Book Review

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A.P. Seyranian and A.A. Mailybaev: "Multiparameter Stability Theory with Mechanical Applications" World Scientific Publishing Co. Pte. Ltd., 2003, ISBN 981-238-406-5

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This recent book by Professor A.P. Seyranian and Dr. A.A. Mailybaev of Moscow State Lomonosov University, Russia, covers fundamental problems, concepts, and methods of multiparameter stability theory with applications in mechanics. It is the main objective of the book to study how stable equilibrium states or steady states of motion may become unstable or vice versa subject to a change of problem parameters. The basic concept is that the parameter space is subdivided into regions of stability and instability, and the authors show that the boundaries between these regions generally consist of smooth surfaces, but that they can have different kinds of singularities. One of the motivations and challenges of the book is to provide the mathematical foundation for bringing qualitative results of bifurcation and catastrophe theory to the space of physical problem parameters with a view to make the theory quantitative as well, and thereby practical and applicable for, e.g. accurate numerical analyses. Thus, the book demonstrates how the stability boundaries and their singularities can be described mathematically by using information of the system at regular or singular points.

A remarkable feature of the book is the authors' presentation of a new multiparameter bifurcation theory of eigenvalues, which is a key point for the stability and instability studies. Two important cases of strong and weak interactions of eigenvalues are distinguished and geometrical interpretations of these interactions are given. The presence of several parameters and the absence of usual differentiability properties for multiple eigenvalues constitute the main mathematical difficulties of the analysis. However, the authors overcome this difficulty by studying bifurcations of eigenvalues along smooth curves in the parameter space emitted from the singular points and then analyzing the obtained relations. With this theory the authors analyze singularities of stability boundaries and present a consistent mathematical description and explanation of several interesting mechanical effects and unexpected phenomena.

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As applications of the presented theory the authors consider a number of mechanical stability and vibration problems including pipes conveying fluid, beams and columns under different loading conditions, rotating shafts and multi-body systems, panels and wings in airflow, etc. For these systems the authors perform their detailed multiparameter stability analysis and show how the developed bifurcation and singularity theory works for specific problems.

The book is based mostly on the authors' personal research carried out in the field during the last 15 years, and may be considered a research monograph. Nevertheless, due to the completeness of the presentation, the book would constitute an excellent textbook for a graduate course on stability theory of mechanical systems. The book is very interesting and gives both a broad and indepth presentation of the subject matter; it is concise and well written, and the material is very well organized.

The book comprises xvi + 403 pages with 144 illustrations, a bibliography of 166 entries of which about 50 cover publications by the authors themselves, and a subject index. The bibliography is a particularly valuable entry to the rich literature published internationally by outstanding Russian researchers in the field, but the reviewer finds that the bibliography could have been more exhaustive.

The material of the book is organized into 12 chapters. Chapter 1, *Introduction to Stability Theory*, provides the general background for stability theory for mechanical systems. Chapter 2, *Bifurcation Analysis of Eigenvalues*, then presents general results of bifurcation theory of simple and multiple eigenvalues of matrices that depend on several parameters. The results of this chapter constitute the main basis for the multiparameter stability analyses and are used in all the subsequent chapters of the book.

Chapter 3, *Stability Boundary of a General System Dependent on Parameters*, is devoted to stability analysis of a general linear system of ordinary differential equations whose coefficients are smooth functions of parameters. Using the results on bifurcation of eigenvalues in the preceding chapter, the authors perform quantitative analyses of the stability domain in the neighbourhood of regular and singular points of the boundary, and thereby derive general and constructive formulas for local approximation of the stability domain using only information at the initial regular or singular boundary points. Chapter 4, *Bifurcation Analysis of Roots and Stability of Characteristic Polynomials Dependent on Parameters*, presents general methods and results of bifurcation analysis of roots of characteristic polynomials associated with linear ordinary differential equations for parameter-dependent mechanical systems, and an expedient approach to the analysis of stability boundaries of the systems is presented.

Chapter 5, *Vibrations and Stability of Conservative Systems*, is concerned with eigenvalue problems for linear conservative mechanical systems, and the changes of multiparameter-dependent simple and multiple eigenvalues are studied. The authors point out that multiparameter stability analysis provides an interesting relationship between singularities of stability boundaries and bimodal solutions in structural eigenvalue optimization problems. This noteworthy feature is exemplified in the book by means of a simple, three-parameter column buckling optimization problem considered by Stephen and William Prager in 1979.

Chapter 6, *Gyroscopic Stabilization*, deals with the stability properties of linear conservative systems with gyroscopic forces, and provides a detailed explanation of the effect of the stabilization in terms of the bifurcation theory of eigenvalues. Thus, it is shown that strong interaction of eigenvalues is the mechanism of gyroscopic stabilization as well as loss of stability. As mechanical examples, stability problems for rotating shafts are considered. Chapter 7, *Linear Hamiltonian Systems*, gives a thorough discussion of Hamiltonian systems with finite degrees of freedom, and it is shown that these systems are characterized by a quite complex set of singularities of the stability boundary. As examples, the stability properties of two gyroscopic systems are investigated: an elastic simply supported pipe conveying fluid, and a rotating multi-body system.

Chapter 8, *Mechanical Effects Associated with Bifurcations and Singularities*, provides detailed investigations of several interesting and surprising mechanical phenomena associated with bifurcations of eigenvalues and singularities of the stability boundary. These phenomena include flutter and divergence instabilities, transition of divergence to flutter, transference of instability between eigenvalue branches, destabilization and stabilization by infinitely small damping, and the disappearance of aeroelastic flutter instability of an unswept wing braced by two different types of struts.

Chapter 9, *Stability of Periodic Systems Dependent on Parameters*, deals with complicated stability problems that have been a challenge for more than one hundred years since the works by Mathieu, Floquet, Hill, Rayleigh, Lyapunov, Poincare, and others. Both in this chapter and in Chapter 10, *Stability Boundary of General Periodic Systems*, the authors give a broad presentation of the application of the new multiparameter stability theory to general periodic systems with analyses of the stability boundaries for the systems.

In Chapter 11, *Instability Domains of Oscillatory Systems with Small Parametric Excitation and Damping*, the authors formulate and solve parametric resonance problems for linear conservative single- and multi-degree of freedom oscillatory systems with the following three physical quantities as parameters: the excitation frequency, the excitation amplitude and the viscous damping coefficient (assuming the two latter parameters to be small). Finally, in Chapter 12, *Stability Domains of Non-Conservative Systems under Small Parametric Excitation*, non-conservative systems are considered and several mechanical examples are presented.

The book is an excellent and most valuable contribution, which I warmly recommend to graduate students and university professors, as well as to researchers and industrial engineers interested in multiparameter stability theory and its applications in mechanics. I expect that this book will serve as an inspiration for studies of new problems, effects, and phenomena associated with instabilities, and that it will provide a new entry to classical problems as well.